Contemporary Calculus - Summer Work

Welcome to Contemporary Calculus! I hope everyone is enjoying his or her summer. This packet is designed for you to practice the mathematical skills that all students are expected to have upon entering this course. Take the time to complete all the problems and make sure you understand how to do the problems. Show all work and answers on a separate sheet of paper. You may research the topics and problems online or at www.Khanacademy.org for additional resources.

This packet will be due <u>the first day of school</u>. The packet will not be graded, but you will be quizzed on the material during the first week of school. I will go over and answer any questions you have about the material.

Exponential and Radical Expressions

Simplify the following expressions.

1)
$$\frac{7x^2}{x^{-3}}$$

2)
$$6y^{-2}(2y^4)^{-3}$$

3)
$$\frac{12(x+y)^3}{9(x+y)^{-2}}$$

4)
$$\frac{3\sqrt{x^3}}{x}$$

Simplify by removing all possible factors from the radical

6)
$$\sqrt[3]{16x^5}$$

7)
$$\sqrt{90(2x-3y)^6}$$

Find the domain of the given expression.

8)
$$\sqrt{5-2x}$$

9)
$$\sqrt{x-1} + \sqrt{5-x}$$

$$10)\,\frac{\sqrt{x-1}}{x-1}$$

Factoring polynomial expressions

Completely factor the following polynomials.

11)
$$8x^4 - 6x^2$$

12)
$$5x^{3/2} + x^{1/2}$$

13)
$$x^2 - 4x + 4$$

14)
$$x^4 - 16$$

15)
$$x^3 - 8$$

16)
$$x^3 - 5x^2 + 5x - 25$$

17)
$$3x(x+1)^3 - 6(x+1)^2$$

Find all real zeros of the polynomial by factoring and setting each factor to zero. Then graph the polynomial and check the zeros on a graphing calculator.

18)
$$x^2 - 5x$$

19)
$$x^4 - 625$$

20)
$$x^2 + x - 20$$

Use the Quadratic Formula to final all real zeros of the second-degree polynomials.

21)
$$x^2 + 6x - 1$$

22)
$$8x^2 - 2x - 1$$

Use synthetic division to complete the indicated factorization (factor completely and find all real zeros).

23)
$$x^3 - 2x^2 - x + 2 = (x+1)$$

Solve the following equations.

$$24) \ 24a - 22 = -4(1 - 6a)$$

$$4 - \frac{1}{x} = \frac{3}{x^2}$$

$$26)\frac{2}{x+5} - \frac{x}{x-5} = 1$$

27)
$$2x^3 + 3x^2 - 8x + 3 = 0$$

Solving Linear and Polynomial Inequalities

Solve the inequality and sketch the graph of the solution on the real number line.

28)
$$x - 4 \le 2x + 1$$

29)
$$-4 < 3x - 5 < 4$$

30)
$$2x^2 - x < 6$$

(Remember: test intervals to determine the solution)

31)
$$\frac{x+5}{x-3} \ge 0$$

Solve the following word problems.

- 32) The revenue for selling x units of a product is R=115.95x, and the cost of producing x units is C=95x+750. To obtain the profit, the revenue must be greater than the cost. For what values of x will the product return a profit?
- 33) A square region is to have an area of *at least* 500 square meters. What must the length of the sides of the region be?

Absolute Value

Solve the following absolute value equations and inequalities.

34)
$$|2x - 5| = 9$$

35)
$$|9 - 2x| < 1$$

36) The American Kennel Club has developed guidelines for judging the various breeds of dogs. For collies, the guidelines specify that the weights for males satisfy the inequality given below, where w is measured in pounds. Determine the interval on the real number line in which these weights lie.

$$\left|\frac{w - 57.5}{7.5}\right| \le 1$$

Rational Expressions

Perform the indicated operations and simplify your answer.

37)
$$\frac{5x+10}{2x-1} - \frac{2x+10}{2x-1}$$

38)
$$\frac{x-1}{x^2+5x+4} + \frac{2}{x^2-x-2} + \frac{10}{x^2+2x-8}$$

Rationalize the denominator.

39)
$$\frac{5}{\sqrt{10}}$$

40)
$$\frac{x}{\sqrt{x-4}}$$

41)
$$\frac{2}{x+\sqrt{3}}$$

40)
$$\frac{x}{\sqrt{x-4}}$$
 41) $\frac{2}{x+\sqrt{3}}$ 42) $\frac{2}{\sqrt{x}+\sqrt{x-2}}$.

Simplify each expression. (Rewrite as one simplified quotient)

43)
$$\frac{-x}{(x+1)^{\frac{3}{2}}} + \frac{2}{(x+1)^{\frac{1}{2}}}$$

44)
$$2\sqrt{x}(x-2) + \frac{(x-2)^2}{2\sqrt{x}}$$

Apply Rational Expressions

45) A retailer has determined that the cost of ordering and storing x units of a product is

$$C = 6x + \frac{900,000}{x}$$

- a) Write the expression of cost as a single fraction.
- b) Determine the cost for ordering and storing 240 units of this product.

Basic Shapes of Curves

- Sketch the graphs on graph paper (if possible). You may use your graphing calculator to verify the graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting at least 3 points, and by your knowledge of transformations.
- List the domain, range, and any asymptotes if they exist.

46)
$$y = \sqrt{x-2}$$

47)
$$y = \ln(x + 2)$$

48)
$$y = -2|x| + 4$$

49)
$$y = \frac{1}{x-2}$$

50)
$$y = \begin{cases} -2x - 3, & x < 0 \\ \frac{x^2 - 9}{x - 3}, & x \ge 0, x \ne 3 \\ 0, & x = 3 \end{cases}$$